Exercice N°1:

1) $|x+3| = 2x-1$

Condition: $2x-1 \geq 0 \iff x \in [\frac{1}{2}, +\infty[$.

Résolution: $|x+3| = 2x-1 \iff$

$x+3 = 2x-1 \quad ou \quad x+3 = -2x+1 \iff x = 4 \quad ou \quad 3x = -2$

$\iff x = 4 \quad ou \quad x = \frac{-2}{3}$

$S_{\mathbb{R}} = \{4\}$

2) $\sqrt{(2x-1)^2} \leq x+1$

Condition: $x+1 \geq 0 \iff x \in [-1, +\infty[$.

Résolution: $(|2x-1|)^2 \leq (x+1)^2$

$\iff (2x-1)^2 - (x+1)^2 \leq 0$

$\iff [2x-1-x-1] \times [2x-1+x+1] \leq 0$

$\iff 3x(x-2) \leq 0$

	$-\infty$	-1	0	2	$+\infty$
$x-2$	$-$		$-$	0	$+$
$3x$	$-$	0	$+$		$+$
$3x(x-2)$	$+$	0	$-$	0	$+$

$$S_{\mathbb{R}} =]-1, +\infty[\cap [0,2] = [0,2]$$

3) $\sqrt{\sqrt{x^2+3}-4} \geq -1$

Condition: $\sqrt{x^2+3}-4 \geq 0$

$\iff \sqrt{x^2+3} \geq 4 \iff x^2+3 \geq 16 \iff x^2 \geq 13$

$\iff (x-\sqrt{13})(x+\sqrt{13}) \geq 0$

	$-\infty$		$-\sqrt{13}$		$\sqrt{13}$		$+\infty$
$x-\sqrt{13}$		$-$		$-$	0	$+$	
$x+\sqrt{13}$		$-$	0	$+$		$+$	
$(x-\sqrt{13})(x+\sqrt{13})$		$+$	0	$-$	0	$+$	

$x \in]-\infty, -\sqrt{13}] \cup [\sqrt{13}, +\infty[$

Résolution: $\sqrt{\sqrt{x^2+3}-4} \geq -1$ toujours vraie

$S_{\mathbb{R}} =]-\infty, -\sqrt{13}] \cap [\sqrt{13}, +\infty[$

4) $\sqrt{2}x^2 + (1-\sqrt{2})x = 1 \iff \sqrt{2}x^2 + (1-\sqrt{2})x - 1 = 0$

$a+b+c = 0 \iff x' = 1 \quad ou \quad x'' = \frac{-1}{\sqrt{2}}$

$S_{\mathbb{R}} = \left\{\frac{-\sqrt{2}}{2}, 1\right\}$

5) $-5x^2 + 6x + 1 = 0$

$\Delta + 36 + 20 + 56$

$x' = \frac{-6-\sqrt{56}}{-10} = \frac{6+\sqrt{56}}{10} \quad ou \quad x'' = \frac{-6+\sqrt{56}}{-10} = \frac{6-\sqrt{56}}{10}$

$S_{\mathbb{R}} = \left\{\frac{6+\sqrt{56}}{10}, \frac{6-\sqrt{56}}{10}\right\}$

Exercice N°2:

1) a) $A(x) = 3x^2 - 21x + 30$

$= 3(x^2 - 7x + 10)$

$= 3[(x - \frac{7}{2})^2 - (\frac{7}{2})^2 + 10]$

$= 3[(x - \frac{7}{2})^2 - \frac{49}{4} + \frac{40}{4}]$

$= 3(x - \frac{7}{2})^2 - \frac{27}{4}$

b) $Min(A(x)) = \frac{-27}{4}; \quad x = \frac{7}{2}$

2) $A(x) = 3((x-\frac{7}{2})^2 - \frac{9}{4})$

$= 3((x-\frac{7}{2})^2 - (\frac{3}{2})^2)$

$= 3(x-\frac{7}{2}-\frac{3}{2})(x-\frac{7}{2}+\frac{3}{2})$

$= 3(x-5)(x-2)$

3) $A(x) = (x-2) \iff 3(x-5)(x-2) = (x-2)$

$\iff (x-2)[(3(x-5)-1] = 0$

$\iff (x-2)(3x-16) = 0$

$\iff x = 2 \quad ou \quad x\frac{16}{3}$

$S_{\mathbb{R}} = \{2, \frac{16}{3}\}$

4) $A(x) = 3x^2 - 21x + 30$

$x' + x" = \frac{-b}{a} = 7$

$x' \times x" = \frac{c}{a} = 10$

$x'^2 + x"^2 = \underbrace{x'^2 + x"^2 + 2x'x"}_{S^2} - 2x'x" = S^2 - 2P = 49 - 2 \times 10 = 29$

Exercice N°3:

1) a) $\overrightarrow{AB}\begin{pmatrix}4\\-3\end{pmatrix}$; $\overrightarrow{AC}\begin{pmatrix}3\\4\end{pmatrix}$

$AB = \sqrt{4^2+3^2} = 5$

$AC = \sqrt{3^2+4^2} = 5$

b) $xx' + yy' = 12 - 12 = 0$.

Par suite, $\overrightarrow{AB} \perp \overrightarrow{AC}$, ainsi $(\overrightarrow{AB}, \overrightarrow{AC})$ est une base orthogonale du plan.

c) $\left.\begin{array}{l} AB = AC \\ \overrightarrow{AB} \perp \overrightarrow{AC} \end{array}\right\} \Longrightarrow ABC$ est un triangle isocèle et rectangle en A.

d) $ABCD$ est un carré $\iff ABC$ est isocèle et rectangle en A et $ABCD$ est un plg.

Donc $ABDC$ est un carré $\iff ABC\overrightarrow{AB} \perp \overrightarrow{A}BDC$ est un plg

$\iff \overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{AD} \iff \begin{pmatrix} x_D - x_A \\ y_D - y_A \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

$\begin{cases} x_D + 4 = 7 \\ y_D = 1 \end{cases} \iff \begin{cases} x_D = 3 \\ y_D = 1 \end{cases} \iff D(3,1)$

2) a) $\begin{cases} \overrightarrow{AB} = 4\vec{i} - 3\vec{j} \\ \overrightarrow{AC} = 3\vec{i} + 4\vec{j} \end{cases}$

$4\overrightarrow{AB}+3\overrightarrow{AC}=25\vec{i} \iff \vec{i} = \frac{4}{25}\overrightarrow{AB}+\frac{3}{25}\overrightarrow{AC}$

$3\overrightarrow{AB}-4\overrightarrow{AC}=-25\vec{j} \iff \vec{j} = \frac{-3}{25}\overrightarrow{AB}+\frac{4}{25}\overrightarrow{AC}$

b) $E(25,-5)$ dans $(O, \vec{i}, \vec{j})$.

$$\begin{aligned} \overrightarrow{OE} &= 25\vec{i} - 5\vec{j} \\ &= 25\left(\frac{4}{25}\overrightarrow{AB}+\frac{3}{25}\overrightarrow{AC}\right) - 5\left(\frac{-3}{25}\overrightarrow{AB}+\frac{4}{25}\overrightarrow{AC}\right) \\ &= \frac{25}{5}\overrightarrow{AB}+\frac{11}{5}\overrightarrow{AC} \end{aligned}$$

Donc $E\left(\frac{23}{5}, \frac{11}{5}\right)$